Name BHAGWAGAR , ID

MTH 213 Discrete Mathematics Summer 2018, 1-3

Solution by Azadan



# Exam I, MTH 213 Discrete Math., Summer 2018

Ayman Badawi

Score = $\frac{\cancel{74}}{76}$

4/4 **QUESTION 1.** (a)( **5 points**) Use the 4-method to convince me that $\sqrt{46}$ is irrational

Assume $\sqrt{46}$ is rational; $\sqrt{46} = \frac{a}{b}$, for some $a, b \in \mathbb{Z}$, $b \neq 0$, $gcd(a,b)=1$

$46 = \frac{a^2}{b^2}$, $gcd(a^2, b^2)=1$; 46 is even $\therefore$ $a^2$ is even & $b^2$ is odd $\therefore$ a is even, b is odd

$\sqrt{46} = \frac{2k}{2m+1}$; for some $k, m \in \mathbb{Z}$; $46 = \frac{(2k)^2}{(2m+1)^2} = \frac{4k^2}{4m^2+4m+1}$; $184m^2 + 184m + 46 = 4k^2$; $46(m^2+m) + \frac{46}{4} = k^2$; non-integer ≠ integer, Contradiction, Assumption invalid $\therefore \sqrt{46}$ is irrational

(b)( **5 points**) Convince me that $\sqrt{65}$ is irrational

Assume $\sqrt{65}$ is rational; $\sqrt{65} = \frac{a}{b}$, for some $a, b \in \mathbb{Z}, b \neq 0, gcd(a,b)=1$;

$65 = \frac{a^2}{b^2}$, $gcd(a^2,b^2)=1$; $b^2 \times 5 \times 13 = a^2$; $5\times13 \mid a^2 \Rightarrow 5\times 13 \mid a$; $a = 5\times13\times k$, for some $k \in \mathbb{Z}$

$b^2 \times 5 \times 13 = (5\times13\times k)^2$; $b^2 \times \cancel{5}\times\cancel{13} = 5^{\cancel{2}}\times13^{\cancel{2}}\times k^2$; $5\times13 \mid b^2 \Rightarrow 5\times 13 \mid b$;

implies $5\times13$ is a factor of both a and b, but $gcd(a,b)=1$; Contradiction, Assumption invalid $\therefore \sqrt{65}$ is irrational //

5/5

(c) ( **5 points**) Let $m$ be an odd number. Prove that $m = 2k-1$ for some integer $k \in Z$.

m is odd, $m = 2p+1$, $p \in \mathbb{Z}$; Let $p = k-1$, $k \in \mathbb{Z}$; $m = 2(k-1)+1$;

$m = 2k-2+1 = 2k-1$ //

5/5

**QUESTION 2.** (a) ( **5 points**) If possible find all solutions of $8x = 12$ over planet $Z_{20}$

$gcd(8,20) = 4$, $4 \mid 12$ ✓ $\therefore$ 4 distinct solutions; $q = \frac{n}{gcd(a,n)} = 5$;

$x_s = 4$; $\therefore x = \{4, 9, 14, 19\}$ //

5/5

(b) ( **3 points**)In view of (a), find all integers, say x, in Planet $Z$ that satisfy $8x \ (mod\ 20) = 12$

Since $q = 5$ and $x_s = 4$;

$x = \{4 + 5k \mid k \in \mathbb{Z}\}$ //

3/3

(c)( **5 points**) Find all integers, say x, in Planet $Z$ that satisfy $3x \ (mod\ 11) = 10$

In $Z_{11}$, $3x = 10$; $gcd(3,11)=1$; $1 \mid 10$ ✓ $\therefore$ 1 distinct solution;

$x = 7$; In $\mathbb{Z}$, $x = \{7 + 11k \mid k \in \mathbb{Z}\}$ //

5/5

(d)( **7 points**) The temperature of a city in Canada is $x$ Celsius, where $-180 < x < 0$ (i.e., x is negative). Given $x \ (mod\ 9) = 4$, $x \ (mod\ 4) = 1$, and $x \ (mod\ 5) = 2$. Find the value of $x$.

$n_1 \quad r_1 \qquad n_2 \quad r_2 \qquad n_3 \quad r_3$

gcd(9,4) = 1 ✓, gcd(9,5) = 1 ✓, gcd(4,5) = 1 ✓ ∴ CRT applies;

$n = 180$; $m_1 = 20$; In $Z_9$, $20x_1 = 1$; $2x_1 = 1$; $x_1 = \{5\}$;

$m_2 = 45$; In $Z_4$, $45x_2 = 1$; $x_2 = 1$; $x_2 = \{1\}$;

$m_3 = 36$; In $Z_5$, $36x_3 = 1$; $x_3 = 1$; $x_3 = \{1\}$;

$x_s = (m_1 n_1 r_1 + m_2 n_2 r_2 + m_3 n_3 r_3) \bmod n = (400 + 45 + 72) \bmod 180 = 517 \bmod 180 = 157$;

$-180 < x < 0$ ∴ $x = 157 - 180 = -23$ //

7/7

(e)( **5 points**) Let $x$ be number of females in MTH 111. Given $x \ (mod\ 7) = 6$, $2x \ (mod\ 10) = 4$. Find all possible values of $x$, where $0 < x < 70$.(Hint: THINK!, it is not difficult)

$x \ (mod\ 7) = 6$; $x \ (mod\ 5) = 2$; gcd(7,5) = 1 ∴ CRT applies

$n = 35$; $m_1 = 5$; In $Z_7$, $5x_1 = 1$; $x_1 = \{3\}$;

$m_2 = 7$; In $Z_5$, $7x_2 = 1$; $x_2 = \{3\}$;

$x_s = (90 + 42) \bmod 35 = 132 \bmod 35 = 27$

$x = 27$ & $27 + 35 = 62$

∴ $x = \{27, 62\}$

$\frac{2x}{10}$ gives q + 4

$\frac{x}{5}$ gives q + 2

5/5

He gave Diff Method which is correct. However, All of you should know the METHOD I EXPLAINED in Class

(f) ( **4 points**) Find $gcd(112, 175)$

$175 \bmod 112 = 63$; $112 \bmod 63 = 49$; $63 \bmod 49 = 14$;

$49 \bmod 14 = 7$; $14 \bmod 7 = 0$; ∴ $gcd(112, 175) = 7$ //

4/4

(g) ( **4 points**) Find $(146)_8 \times (54)_8$

$$\begin{array}{r} (146)_8 \\ \times (54)_8 \\ \hline (630)_8 \\ + (7760)_8 \\ \hline (10610)_8 \end{array}$$

4/4

(i) ( **4 points**)Convert 317 to base 16

317 div 16 = 19; 19 div 16 = 1; 1 div 16 = 0.

317 mod 16 = 13 = D; 19 mod 16 = 3; 1 mod 16 = 1

∴ $(317)_{10} = (13D)_{16}$ //

4/4

**QUESTION 3.** (a)( **4 points**) Let $x, y, z$ be strings of binary codes, $x = 11011$, $y = 10110$, and $z = 01110$. Find $(x \oplus z) \vee y$

$x = 11011$
$z = 01110$
$y = 10110$
$(x \oplus z) \vee y = 10111$ //

4/4

(b) ( **6 points**) Let $S_1, S_2, S_3$ be some statements. Use truth table and convince me that

$$(S_1 \wedge S_2) \Rightarrow S_3 \equiv (S_1 \Rightarrow S_3) \vee (S_2 \Rightarrow S_3)$$

6/6 $(S_1 \wedge S_2) \Rightarrow S_3$ identical, ∴ $(S_1 \Rightarrow S_3) \vee (S_2 \Rightarrow$

| $S_1$ | $S_2$ | $S_3$ | $S_1 \wedge S_2$ | $S_1 \Rightarrow S_3$ | $S_2 \Rightarrow S_3$ | $(S_1 \wedge S_2) \Rightarrow S_3$ | $(S_1 \Rightarrow S_3) \vee (S_2 \Rightarrow S_3)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

(c) ( **5 points**) Let $S_1, S_2$ be some statements. Is the expression $[(S_1 \Rightarrow S_2) \wedge \neg S_2] \Rightarrow \neg S_1$ a tautology? Explain using truth-table.

not identical ∴ ~~Not~~ a tautology

| $S_1$ | $S_2$ | $S_1 \Rightarrow S_2$ | $\neg S_2$ | $(S_1 \Rightarrow S_2) \wedge \neg S_2$ | $\neg S_1$ | $[(S_1 \Rightarrow S_2) \wedge \neg S_2] \Rightarrow \neg S_1$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |

Tautology.

−2

(d) Write down T or F ( **9 points**)

True

(i) $\exists!\, x \in N^*$ such that $\forall\, y \in Q$, we have $xy - 2y = 0$ T

(ii) $\forall\, x \in Q, \exists!\, y \in N^*$ such that $yx - 5x = 0$ F, if $x = 0$, then y is not unique.

(iii) $\forall\, y \in R^*, \exists!\, x \in R^*$ such that $yx - y^2 = 0$ T

true! $x \neq 0$ false

(iv) If $\exists!\, x \in N^*$ such that $x^2 - 4x = 0$, then $4x - 1 = 12$ F

True $x = \pm 2$ False

(v) If $\exists\, x \in Z$ such that $x^2 = 4$, then $x^3 = 8$ F, when $x = -2$, $x^3 = -8$, not 8.

$x = 1$ true $y = 0$ False

(vi) If $\exists!\, x \in N^*$ and $\exists!\, y \in N$ such that $x^2 + y^2 = 1$, then $y - x = x - y$ F, $x = 1$, $y = 0$, $y - x = -1$
$x - y = 1$

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$x(x-4) = 0$

**QUESTION 4. (6 points)**
Use math induction to convince me that $12 \mid (5^{4m} - 1)$ , $m \geq 1$.

$\boxed{1}$ Prove it for $m = 1$:

when $m = 1$:

$5^{4(1)} - 1 = 5^4 - 1$
$= 625 - 1$
$= 624$

$\frac{624}{12} = 52$.

$\therefore 12 \mid 5^4 - 1$.

$\boxed{2}$ Assume claim is valid for some $n = m \geq 1$:

$\Rightarrow 12 \mid 5^{4n} - 1$

$\boxed{3}$ Prove it for $(n+1)$:

[ie. we must show that $12 \mid 5^{4(n+1)} - 1$].

$5^{4(n+1)} - 1 = 5^{4n+4} - 1$
$= 5^{4n} \cdot 5^4 - 1$
$= 5^{4n} \cdot 5^4 - 5^4 + 5^4 - 1$
$= 5^4 \underbrace{(5^{4n} - 1)}_{\text{Divisible by 12, as shown in } \boxed{2}} + \underbrace{5^4 - 1}_{\text{Divisible by 12, as shown in } \boxed{1}}$.

$\therefore 12 \mid 5^4(5^{4n} - 1) + 5^4 - 1$.

ie. $12 \mid 5^{4(n+1)} - 1$.    Hence $12 \mid 5^{4m} - 1$. QED.

**QUESTION 5. (5 points)** Let $A = \{\{3\}, 3, \cancel{3}, 5, \{3,5\}, \{\phi\}, \{6\}, \{6, x\}, x, 7\}$ and $B = \{3, \{3,5\}, x, \{7\}, \{3\}\}$. Then Write T or F

(i) $\{3\} \in A \cap B \longrightarrow$ T.

(ii) $7 \in A - B \longrightarrow$ T.

(iii) $\{\phi\} \in A - B \longrightarrow$ T.

(iv) $\{\phi\} \subset A - B. \longrightarrow$ F. [$\{\{\phi\}\} \subset A - B$ would be true].

(v) $\{3, \phi\} \subset A \longrightarrow$ F. $\phi \not\subset A$

(vi) $\{3,5\} \in A \longrightarrow$ T.

(vii) $\{3,5\} \subset A \longrightarrow$ T.

(viii) $B - A = \phi \longrightarrow$ F. $B - A = \{\{7\}\}$.

(ix) $|AXB| = 13 \longrightarrow$ F.

(x) $\{\{3\}, x\} \subset A \longrightarrow$ T.

~~AXB~~ $|AXB| = |A| \times |B| = 9 \times 5 = 45$

**QUESTION 6.** **(8 points)** Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. Define $=$ on A, where if $a, b \in A$, then $a = b$ if $(a + (12 - b)\ (mod\ 12) \in \{0, 4, 8\}$

(i) Convince me that $=$ is an equivalence relation on $A$.

Since 12 (mod 12) = 0, obseve tht (a + 12 - b) (mod 12) = (a - b) ( mod 12). Let B = {0, 4, 8}

Remark: (1) note that (c + d) (mod 12) is in B for every c, d in B.

(2) note that in general If L (mod n) = k and k not = 0, then -L (mod n) = n - k

(A-A): Let a in A. Then (a - a) ( mod 12) = 0 in B. Thus a = a for every a in A

(A-B): Assume that a = b for some a, b in A. We show b = a. We may assume that a, b are different elements in A. Since a = b and b is not equal to a, (a - b) ( mod 12) = 4 or 8. If (a - b) (mod 12) = 4, then (b - a)(mod 12) = 12 - 4 = 8 in B (see 2 above). Thus b = a. If (a - b) (mod 12) = 8, then (b - a) (mod 12) = 12 - 8 = 4 in B. Hence, again, b = a.

(A-B-C): Assume that a = b and b = c for some a, b, and c in A. Hence (a - b) (mod 12) is in B and (b - c) (mod 12) is in B. Let n = (a - b) ( mod 12) and m = (b - c) mod (12). Note that n, m are in B. Then (a - c) (mod 12) = [(a - b) + (b - c)] (mod 12) = (n + m) (mod 12) is in B by (1). Thus a = c

$(a + 12 - b + b + 12 - c)(mod\ 12)$

$(1 + 12 - 5 + 5 + 12 - 9)(mod\ 12) = 16$

$16\ (mod\ 12) = 16$

$(16 - 16)(mod\ 12) = 0.$

$(0)(mod\ 12) = 0.$

$0 \in \{0, 4, 8\}$. $\therefore$ a "=" c.

$\therefore$ Axiom 3 holds.

$\therefore$ "=" is an equivalence relation.

(ii) Find all equivalence classes of $(A, =)$.

$[0] = \{0, 4, 8\}$.

$[1] = \{1, 5, 9\}$.

$[2] = \{2, 6, 10\}$.

$[3] = \{3, 7, 11\}$.

(iii) view $=$ as a subset of $A \times A$. How many elements does $=$ have? Do not write down all elements of $=$

"=" $= ([0] \times [0]) + ([1] \times [1]) + ([2] \times [2]) + ([3] \times [3])$.

$= 3^2 + 3^2 + 3^2 + 3^2$

$= \underline{\underline{36}}$.

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# Exam I: MTH 213, Spring 2018

Tasneem Batool

Ayman Badawi

Score = $\frac{64}{64}$

**QUESTION 1.** (i) **(5 points)** Prove that $\sqrt{55}$ is irrational. (Hint: You must use this technique: Deny. Then $\sqrt{55} = a/b$ for some positive ODD integers a, b s.t. $gcd(a,b) = 1$, now start cooking as explained in the class)

Deny; say $\sqrt{55}$ is rational.

$\sqrt{55} = \frac{a}{b}$, $a, b \in \mathbb{Z}$, $b \neq 0$, $\gcd(a,b) = 1$.

a and b are odd integers, let $a = 2m+1$; $b = 2n+1$., $m, n \in \mathbb{Z}$.

$\sqrt{55} = \frac{2m+1}{2n+1}$

$55 = \frac{4m^2+4m+1}{4n^2+4n+1}$

$55(4n^2+4n+1) = 4m^2+4m+1$

$55(4n^2+4n)+54 = 4m^2+4m$

$\underbrace{55n^2 + 55n + \frac{54}{4}}_{\notin \mathbb{Z}} = \underbrace{m^2+m}_{\text{integer.}}$ Contradiction. Hence $\sqrt{55}$ is irrational.

5/5

(ii) **(3 points)** Prove that $\sqrt{5} + \sqrt{11}$ is irrational (Hint: You may use the result from (i))

Deny. Say $\sqrt{5} + \sqrt{11}$ are rational.

$\sqrt{5} + \sqrt{11} = \frac{a_0}{b_0}$ $a_0, b_0 \in \mathbb{Z}$, $b_0 \neq 0$, $\gcd(a_0, b_0) = 1$.

$(\sqrt{5}+\sqrt{11})^2 = \left(\frac{a_0}{b_0}\right)^2$

$5 + 2\sqrt{55} + 11 = \frac{a_0^2}{b_0^2}$

$\sqrt{55} = \frac{a_0^2}{2b_0^2} - \frac{16}{2}$. LHS is irrational as shown in (i). RHS is rational. Contradiction. Hence, $\sqrt{5}+\sqrt{11}$ is irrational.

3/3

**QUESTION 2.** (i) **(6 points)** For every $n \geq 1$, use math induction to prove that $18 \mid (5^{6n} - 1)$.

$\boxed{1}$ Prove for $n=1$.

$5^6 - 1 = 15624$, $18 \mid 15624$.

$\boxed{2}$ Assume: $18 \mid 5^{6n} - 1$ for some $n \geq 1$

$\boxed{3}$ Prove for $n+1$.

$5^{6n+6} - 1$

$= 5^{6n} \cdot 5^6 - 1$

$= 5^{6n} \cdot 5^6 - 5^6 + 5^6 - 1$

$= \underbrace{5^6(5^{6n}-1)}_{\text{divisible by 18, as shown in } \boxed{2}} + \underbrace{5^6 - 1}_{\text{divisible by 18, as shown in } \boxed{1}}$

6/6

Hence $18 \mid 5^6(5^{6n}-1) + 5^6 - 1 \Rightarrow 18 \mid 5^{6n} - 1$.

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(ii) **(3 points)** Use direct proof to show that $18 \mid (5^{6n} - 1)$, for every $n \geq 1$.

$18 = 2 \times 3^2$, $\phi(18) = 1 \times 2 \times 3 = 6$. $\gcd(5, 18) = 1$.

By Euler Fermat result, $5^6 \equiv 1 \pmod{18}$

Multiplying $5^6$ $n$ times: $(5^6)^n \equiv 1^n \pmod{18}$

$5^{6n} \equiv 1 \pmod{18}$.

Hence $18 \mid 5^{6n} - 1$.

**QUESTION 3.** (i) **(5 points)** Use math induction to prove that $\sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} = \frac{n+1}{4n+20}$ for every $n \geq 1$

$\boxed{1}$ Prove for $n=1$.

$\sum_{i=0}^{1} \frac{1}{(i+4)(i+5)} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12}$. check $\frac{n+1}{4n+20} = \frac{2}{24} = \frac{1}{12}$

$\boxed{2}$ Assume: $\sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} = \frac{n+1}{4n+20}$ for some $n \geq 1$.

$\boxed{3}$ Prove for $n+1$.

better if you write we show $\sum_{i=0}^{n+1} \frac{1}{(i+4)(i+5)} = \frac{n+2}{4n+24}$

$\sum_{i=0}^{n+1} \frac{1}{(i+4)(i+5)} = \sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} + \frac{1}{(n+5)(n+6)}$

$= \frac{n+1}{4n+20} + \frac{1}{(n+5)(n+6)}$

$= \frac{n+1}{4(n+5)} + \frac{1}{(n+5)(n+6)}$

$= \frac{(n+1)(n+6) + 4}{4(n+5)(n+6)} = \frac{n^2 + 7n + 10}{4(n+5)(n+6)} = \frac{(n+2)\cancel{(n+5)}}{4\cancel{(n+5)}(n+6)} = \frac{n+2}{4n+24}$

5/5

$\sum_{i=0}^{n+1} \frac{1}{(i+4)(i+5)} = \frac{(n+1)+1}{4(n+1)+20}$, hence $\sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} = \frac{n+1}{4n+20}$, $\forall n \geq 1$.

(ii) **(3 points)** Use direct proof to show that $\sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} = \frac{n+1}{4n+20}$ (Hint: First note that $\frac{1}{(i+4)(i+5)} = \frac{1}{i+4} - \frac{1}{i+5}$. For each $0 \leq i \leq n$, let $a_i = \frac{1}{i+4} - \frac{1}{i+5}$. Now calculate $a_0 + a_1 + \cdots + a_n$ and stare, you should observe something.!)

$\sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} = \sum_{i=0}^{n} \left[\frac{1}{(i+4)} - \frac{1}{(i+5)}\right] = a_0 + a_1 + \dots a_n$ where $a_i = \frac{1}{(i+4)} - \frac{1}{(i+5)}$

$\sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} = \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} + \frac{1}{6} - \frac{1}{7} \dots + \frac{1}{n+3} - \frac{1}{n+4} + \frac{1}{n+4} - \frac{1}{n+5}$

telescopic?

Note all terms except first and last cancel out

$\sum_{i=0}^{n} \frac{1}{(i+4)(i+5)} = \frac{1}{4} - \frac{1}{(n+5)} = \frac{n+5-4}{4(n+5)} = \frac{n+1}{4n+20}$.

3/3

**QUESTION 4.** **(3 points)** Find $(265)_7 \times (56)_7$

$$\begin{array}{r} 2\,6\,5 \\ \times\quad 5\,6 \\ \hline 2\,3\,5\,2 \\ +\,2\,0\,5\,4\,0 \\ \hline (2\,3\,2\,2\,2)_7 \end{array}$$

(mod 7) (div 7)

Ans: $(23222)_7$

3/3

**QUESTION 5.**

**(3 points)** Find $(1055)_9 - (338)_9$

$$\begin{array}{r} 1\,0\,5\,5 \\ -\quad 3\,3\,8 \\ \hline 6\,1\,6 \end{array}$$

(mod 9) (div 9)

Ans: $(616)_9$

3/3

**QUESTION 6. (4 points)** JUST WRITE T OR F

(i) $\exists! x \in Z$ such that $\forall y \in R$, $x + y = y$ T

(ii) $\forall x \in Z_6^*$, $\exists y \in Z_6^*$ such that $xy = 1$ over planet $Z_6$ (note $Z_6^* = \{1, 2, 3, 4, 5\}$) F

(iii) $\forall x \in Z_6^*$, $\exists! y \in Z_6^*$ such that $xy = 1$ over planet $Z_6$ (note $Z_6^* = \{1, 2, 3, 4, 5\}$) F

(iv) $\exists! x \in Q^*$ such that $2x^2 + 3x + 1 = 0$ F not unique

**QUESTION 7. (8 points)** Let $d = gcd(98, 119)$. Find $d$ over PLANET $N$. Then find two integers in PLANET $Z$, say $m, n$, such that $d = 98n + 119m$.(Show the work)

gcd (98, 119)

$119 = 1 \cdot 98 + 21$ $\rightarrow$ $98 = 4 \cdot 21 + 14$ $\rightarrow$ $21 = 1 \cdot 14 + 7$ $\rightarrow$ $14 = 2 \cdot 7 + 0$

$\gcd(98, 119) = 7 \in N$

$$\begin{aligned}
7 &= 21 - 14 \\
&= 21 - (98 - 21(4)) \\
&= 21 - 98 + 21(4) \\
&= 5(21) - 98 \\
&= 5(119 - 98) - 98 \\
&= 5(119) - 5(98) - 98 \\
7 &= 5(119) - 6(98)
\end{aligned}$$

$n = -6 \quad m = 5$ , $n, m \in Z$.

8/8

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[redacted]

**QUESTION 9.** (i) **(5 points)** Solve $6x = 3$ over planet $Z_9$.

$6x = 3$ in $Z_9$ $\gcd(6,9) = 3$ $\cancel{3 \nmid}$ $3 \mid 3$.

$\therefore$ 3 sol$^{\circ}$.

$x = 2$, $x = 5$, $x = 8$

(ii) **(3 points)** Solve over planet $Z$, $6x \equiv 3 \ (mod\ 9)$

$x = 2 + 9k_1$, $x = 5 + 9k_2$, $x = 8 + 9k_3$

$k_1, k_2, k_3 \in \mathbb{Z}$.

**QUESTION 10. (8 points)** Let $X$ be the number of females in some sport-activity at the AUS. Given $X \equiv 2 \ (mod\ 4)$, $X \equiv 5 \ (mod\ 9)$, and $X \equiv 10 (mod\ 11)$. If $0 < X < 396$, then find $X$. (Show the work)

$\underset{r_1}{x} \equiv 2 \ (\text{mod } \underset{m_1}{4})$ $\underset{r_2}{x} \equiv 5 \ (\text{mod } \underset{m_2}{9})$ $\underset{r_3}{x} \equiv 10 \ (\text{mod } \underset{m_3}{11})$

$(m_2 m_3)^{-1} \text{ mod } m_1$ $(m_1 m_3)^{-1} \text{ mod } m_2$ $(m_1 m_2)^{-1} \text{ mod } m_3$

$99x = 1$ in $Z_4$ $44x \equiv 1 \ (\text{mod } 9)$ $36x \equiv 1 \ (\text{mod } 11)$

$3x = 1$ in $Z_4$ $8x \equiv 1 \ (\text{mod } 9)$ $3x \equiv 1 \ (\text{mod } 11)$

$\boxed{x = 3 = d_1}$ $\boxed{x = 8 = d_2}$ $\boxed{x = 4 = d_3}$

$X = 99(3)(2) + 44(8)(5) + 36(4)(10)$

$= 3794 \text{ mod } 396$

$X = 230$, $0 < x < 396$.

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